national accelerator laboratory

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Subject: RF CONSIDERATIONS FOR 100 GeV INTERSECTING STORAGE RINGS

1. Assumptions

We assume the storage ring to have an average radius of 1/3 km and a transition energy given by $\gamma_t = 15$. Since the beam is to be transferred to the storage rings without any intermediate debunching and rebunching process the rf frequency is the same as in the main ring viz. 53.4 MHz. Thus, the harmonic number, h, equals 373.

We assume the phase-plane area of one bunch - in a $\Delta\ (\beta\gamma),\ \phi_{rf}$ plane - to be given by

$$A = 5.5 \times 10^{-2} \text{ rad.}$$

This is the area given in* the booster parameter list of June 25, 1968, for the booster injection point, times an assumed blow-up factor of 8. The exact value of the blow-up, to be expected mainly at the two transition energies and at the booster to main-ring beam transfer, is not known at this point. This is unfortunate since most rf parameters depend strongly on the value of A. At least, scaling to other values of A is straightforward. It is felt that the assumed blow-up factor of 8 is rather safe, if not slightly pessimistic.

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* The figure given there - 1.6 eVs - is the total area of all 84 booster bunches in a ΔE, Δt plane.

2. Parameters during stacking

The number of main-ring pulses to be stacked is expected to be rather small (at the assumed value of A about 20 pulses will lead to a stacked current of 10^{15} protons in a $\pm 10^{-3}$ momentum spread). Hence a low value of $\Gamma = \sin \phi_s$ has to be chosen to yield a reasonably good approximation to a rectangular distribution of particle density versus momentum. The choice of $\Gamma = 0.5$, combined with instantaneous rf turn-off, seems to be a reasonable one.

When the stacking buckets start to enter the phase-space already occupied by previously stacked particles, the buckets have to fit tightly around the bunches in order to achieve maximum stacking efficiency. Hence, from this moment on at least, the bucket area must be equal to A. The corresponding rf voltage is given by

$$eV_0 = \frac{E_0 A^2 \pi h \xi}{128 \gamma \alpha^2} . \qquad (1)$$

Where E_0 is the particle rest energy, γ the total energy over E_0, ξ stands for

$$\left| \frac{1}{\gamma^2} - \frac{1}{\gamma_t^2} \right| , \qquad (2)$$

and the bucket-area parameter α is a function of Γ only and equals 0.333 for $\Gamma = 0.5$. For the parameters chosen one finds

$$\xi = 4.4 \times 10^{-3} ,$$

and

$$V_0 = 10 \text{ kV}.$$

This is rather convenient and in contrast to the inconveniently low voltages required in the CERN IRS (due to the low rf frequency imposed by the CPS and the small phase-plane area obtained in that machine at present intensities). It should be noted though, that V scales with A^2.

On the other hand, it should be noted that 10 kV is probably too large to make the "missing bucket" scheme possible, so that it is probably excluded to handle fractional-turn injection situations like the ones occurring in the CERN ISR.

The rate of acceleration with bucket-area A is given by

$$\frac{\dot{p}}{p} = \frac{A^2 \Gamma h \xi c}{256 \gamma^2 \alpha^2 \beta R}, \qquad (3)$$

where p is the particle momentum, βc its velocity and R the average radius.

Assuming that the maximum distance between the injection orbit and the lower edge of the stack is 2% in $\Delta p/p$ one finds that it takes

2.6 s

at $\Gamma = 0.5$ to accelerate the beam across that distance. The time for going across a stack of a few times 10^{-3} width is correspondingly smaller.

Hence, it would be possible, in principle, to employ a fully repetitive stacking scheme with a constant voltage of 10 kV from beginning to end of the stacking cycle and constants turn-off frequency, corresponding to the desired position of the upper edge of the stack, without exceeding the main-ring cycle time. However, a larger voltage is very desirable at injection (cf. the next section) and is also useful for reducing the time for accelerating from the injection orbit to the vicinity of the stack. If a larger voltage is used it has to be adiabatically reduced to the value of 10 kV corresponding to full buckets. This is the same scheme that is used in the CERN ISR but the factor by which the voltage has to be modulated is much smaller and there is no difficulty in conducting the process sufficiently rapidly. The phase oscillation frequency corresponding to the final voltage is given by

$$f_p = \frac{A L \xi c}{32 \pi \alpha \gamma R} \sqrt{\cos \phi_s} \qquad (4)$$

and amounts to about 20 Hz for the parameters chosen.

3. Matching

The bunches in the main ring are too wide in momentum and too short in phase to match the shape of storage ring buckets of area A. In principle one could accomplish all or part of the matching in the main ring, either by an adiabatic reduction of rf voltage or by one of the well-known non-adiabatic methods. It seems desirable, however, not to load the main-ring with any such tasks since they may interfere with other simultaneous uses of the main-ring and may require ejection from a flat top at 100 GeV that is not otherwise required. Instead it seems preferable to carry out all the matching by rf manipulations in the storage ring. This requires, however, that the original main-ring bunches fit into the linear region of the storage-ring buckets. We ask, therefore, that the width of the storage-ring buckets at injection equals twice the width of the main-ring bunches. We assume $\Gamma = 0$ for this case since this can easily be arranged at injection.

For the assumed value of A and 3.5 MV per turn, $\phi_s = 45°$ in the main ring one finds a bunch width given by*

$$\Delta(\beta\gamma)_e = \pm 4.7 \times 10^{-2}.$$

The voltage required to provide a bucket width equal to twice that value at $\Gamma = 0$ equals

$$\left[16\,\alpha\,\frac{\Delta(\beta\gamma)_e}{A}\right]^2 V_0 , \qquad (5)$$

and amounts to

200 kV

for our parameters.

We propose building an rf system capable of delivering this voltage. The relatively high voltage capability will be useful in other respects too, e.g. for acceleration or deceleration.

A possible matching scheme is the following:

i) have constant rf voltage from injection onwards,

ii) wait till 1/8 phase oscillation after injection,

iii) switch rf phase by π, so as to center the bunch at the antistable point,

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* This is larger than the value scaled from the momentum spread at 200 GeV given in the NAL parameter list, probably because of a larger

iv) wait till bunch has been deformed into the desired shape in phase-space,

v) switch back to original phase.

Another scheme could be

i) wait 1/4 phase oscillation at full voltage,

ii) reduce voltage rapidly, to match the bunch which is now narrow in momentum and long in phase,

iii) increase voltage adiabatically to value desired for acceleration.

In both cases it is assumed that the bunches are being centered in the storage ring buckets by a fast and precise phase-lock beam-control system. This will not be difficult so long as one does not attempt to combine the bunches of subsequent main-ring acceleration cycles as would be required for injecting more than 3 turns in transverse phase space.

The transition from $\Gamma = 0$ at injection to finite acceleration rate, say $\Gamma = 0.5$, must be made adiabatically, but this does not present any difficulties.

If one wanted to reduce the size and cost of the storage-ring rf system to the absolute minimum possible one would have to load the main ring rf system with the task of matching in longitudinal phase space. Even then, one would have to incorporate some margin of bucket-area in the storage ring, at least a factor of two, say, for $\Gamma = 0.5$.

This would bring the storage ring peak rf voltage to 40 kV (instead of 200).

If the matching to this storage ring voltage were done by an adiabatic reduction of the main-ring voltage this reduction would have to be from 3.5 MV to 70 kV in about 50 ms, in the presence of about 400 m A (average current) beam loading.

4. RF Cavities

In principle it is possible to generate the recommended voltage of 200 kV (peak) in a single cavity. This does not seem advisable, however, in view of the fact that the following conditions should be met:

i) The rf amplifier has to cover a rather large dynamic range ($>$ 20 to 1 for the parameters chosen).

ii) The cavity has to present a low impedance to the beam at the fundamental bunch frequency. As a rough estimate, one may say that the voltage induced by the peak rf beam current of 2.4 A should not exceed a certain fraction, say 1/10 of V_0. This suggests a maximum total impedance of the order of 4 kΩ which may require a feedback system, similar to the one used in the CERN ISR.

iii) The cavity must have a low (or at least well controlled) impedance for harmonics of the bunch-frequency (or perhaps even of the revolution frequency). This requires the cavity to be free of spurious higher order resonances up to several hundred megahertz.

These requirements can be met more easily if one employs several cavities of correspondingly lower voltage. As a first approach to a more detailed design, one may consider ten cavities, say, delivering 20 kV peak voltage each.